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Instructional Recommendations for Elementary and Intermediate Mathematics Instruction

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During the process of reviewing student work from previous administrations of the New York State elementary and intermediate mathematics assessments, a number of themes and trends emerged. The following recommendations will help guide teachers of mathematics as they plan instruction based on the New York State Mathematics Core Curriculum (Revised 2005)

The Mathematics Toolkit contains curriculum guidance materials and resources that support the New York State Learning Standard for Mathematics to assist educators in developing mathematics lessons and activities.

Reading in Mathematics

The New York State Learning Standards for Mathematics requires students to read and comprehend mathematical problems in context. It is imperative that teachers provide students opportunities to solve problems that are of a contextual nature and practice with their students the process of "pulling apart" the "given" from the "must find".

For example, teachers should instruct students to use broad understanding in problem solving, rather than keywords to determine operations. Words such as *and, all, left, all together,* etc. have multiple meanings and therefore students who use keywords to determine the operation(s) may select the wrong operation.

Example: Mardio is trying to collect an entire set of 12 toy cars. He already has 4 of the cars. How many *more* does he need?

This problem requires a comparison of two quantities, how much bigger is 12 than 4? The word *more* is often suggested as a keyword for addition, but this problem cannot be solved by adding 12 and 4.

Vocabulary

Appropriate mathematical vocabulary should be used during instruction. For example, students should be taught the difference between a bar graph and a histogram. (See Glossary for Teachers and Suggested Lists of Mathematical Language by Grade Level.)

Mathematical Language

The Glossary, intended for teacher use only, provides an understanding of the mathematical terms used

in Pre K-grade 8 level instruction, and in Regents-approved courses entitled Integrated Algebra, Geometry, and Algebra 2 and Trigonometry as reflected in the New York State Mathematics Core Curriculum (Revised 2005). We encourage all teachers to become familiar with these terms and use them consistently throughout a student's educational program. For example, students should be taught the difference between an expression and an equation. (See Glossary for Teachers and Suggested Lists of Mathematical Language by Grade Level.)

The Suggested Lists of Mathematical Language are presented for each Pre K-grade 8 level instruction, and for Regents-approved courses entitled Integrated Algebra, Geometry, and Algebra 2 and Trigonometry. These lists are intended to engage New York State educators and students in building a mathematical language. The lists contain terms that are defined in the Glossary. (See Glossary for Teachers and Suggested Lists of Mathematical Language by Grade Level.)

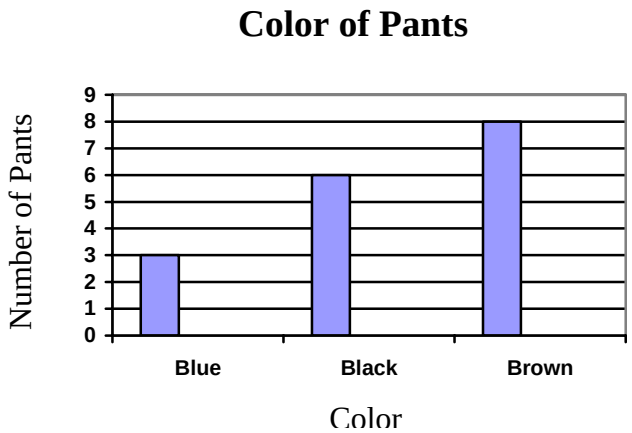
Organizing Work
Students need to be taught how to organize their work. Their responses should include tables, charts, graphic organizers, diagrams and lists, where appropriate. Students should document their work by showing their procedures on paper and always showing the last step of calculations. Students may use trial and error but by 6th grade, they should understand the need to always show at least three trials, one below and one above the chosen answer in order to support their logic.

Scoring with Holistic Rubrics
Good instructional practice includes evaluations with holistic rubrics that capture the overall quality of student performance. A holistic rubric generally specifies several levels (each falling within an interval along a scale) of overall performance along with a list of features that characterize each level. In holistic scoring, a numerical rating is assigned based on the work as a whole.

New York State mathematics assessments at the elementary and intermediate levels use 2-point and 3-point holistic rubrics. The holistic rubrics should be posted in the classroom, and students should use these rubrics throughout the year to assess their own work and the work of others.

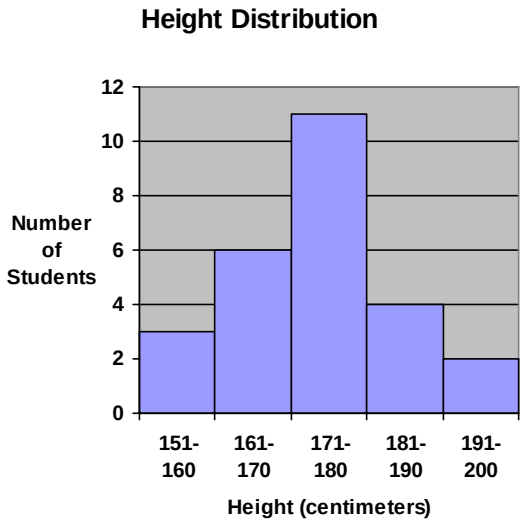
Graphing
Third and fourth grade students must be able to read, interpret and/or construct bar graphs and pictographs, as well as read and interpret line graphs. Eighth grade students must be able to read, interpret, and/or construct bar graphs, line graphs, double-line graphs, histograms, and circle graphs. In graphing, students must provide a title for the graph, label the axes, use an appropriate scale (starting at zero and labeled at intervals not between intervals) and graph all the data as in the bar graph diagram below.

A **bar graph** is used to represent discrete qualitative data such as eye color or height and is therefore not placed on a continuum. It is used to compare single events and the differences between them, using a bar for each data set and groups data together according to categories, displaying a bar for each value in that category. Bar heights represent quantities in a set of data. Bar graphs can be used to compare data sets against some variable data such as time or frequency. The bars in a bar graph are placed at a uniform distance from each other.



A **histogram** is used to represent quantitative data such as test scores and must, therefore, be placed on a continuum with touching bars in order to display the data sequentially in intervals. Histograms are constructed based on data in a frequency table where the categories are consecutive intervals along the continuum or numeric scale. The intervals (and thus the width of each bar) should always be the same size with no gaps between them. The number of data elements falling within a particular interval determines the height of each bar. Questions that should be asked:

- What is the appropriate interval to use for the bar width?
- What is a good scale to use for the height of the bars?



A **number sentence** is horizontal and has three parts: a left-hand side, a relation symbol, and a right-hand side. In the number sentence, 4 + 5 = 9, "4 + 5" is the left-hand side, "=" is the relation symbol, and "9" is the right-hand side. A number sentence in which the relation symbol is the equal sign (=) is called an **equation**. A number sentence in which the relation symbol is $<$, $\leq$, $>$, $\geq$, or $\neq$ is called an **inequality**. A number phrase such as 22 + 34, is an **expression**. A number phrase with a variable but no relation symbol such as x + 4, is called an **algebraic expression**.

Calculators
Appropriate grade-level calculators should be made available to students in the classroom. They may be used to assist students in their understanding of concepts and procedures and to facilitate cumbersome computations. The use of calculators should not be a substitute for a student's understanding of quantitative concepts and relationships or proficiency in basic computations. When using a calculator, students should document their work by showing their procedures on paper and always showing their last step of calculations.

Further information on the acceptable use of technology in instruction and assessment is available at Guidance for Calculator Use in the Classroom and on State Assessments in Mathematics.

Pi
Students should learn that π is an irrational number and, unless otherwise specified, the π key and the full display of the calculator should be used in computations. π is not equal to 3.1416, 3.14 nor $\frac{22}{7}$.

When working without a calculator, students should leave their answers in terms of π for greatest accuracy.

Diagrams Drawn to Scale
On the New York State mathematics assessments all diagrams are drawn to scale, unless labeled "not to scale." If a diagram is labeled "not to scale" then measurement devices may **not** be used to solve the problem.

Rounding refers to the process of making a reasonable approximation of a number. Rounding is done at the **end** (for example, round your **answer** to the nearest tenth).

Reasonableness is dictated by a variety of factors.

Estimating is a technique used to facilitate calculation of cumbersome or 'awkward' numbers. Each awkward number is rounded **before** performing the specified operation.

In that we want our estimate to fall as close to the actual answer as possible, with a series of numbers, it is **not** necessary to round all of the numbers (unless instructed to do so), only the cumbersome ones.

The final answer in an estimate is **not** rounded since this would result in an answer even further from the actual answer.

Equivalent Forms
If a specific form is **not** required for an answer, a student may state the answer in any equivalent form.
Examples:

- Probability can be expressed as a fraction, decimal, percent, ratio or where *m* and *n* are integers: *m* to *n*; *m* in *n*; *m* chances out of *n*; *m* chances in *n*; and if equal probability, equally likely.

- Ratios can be expressed in a variety of ways, such as *a* to *b*, *a:b*, or $\frac{a}{b}$.

Rules and Functions

Bead Designs

Number of Designs (x)	Total Number of Beads (y)
2	12
3	18
4	24
5	
Write the rule that can be used to find the number of beads used to make any number of designs.	

A rule or a function identifies the relationship between two variables. The rule for the information in the chart above would be: 6 times the number of designs equals the number of beads or 6x = y.

Mathematical Equations

When solving problems involving multiple steps, focus students' attention on writing each step using a mathematically correct equation. For example:

Mathematically Correct Equations:

$15 \div 3 = 5$
and
$5 + 1 = 6$
and
$6(7) = 42$

Mathematically Incorrect Expression or "Run on Equation":

$15 \div 3 = 5 + 1 = 6(7) = 42$